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Interaction Theory of the Electromagnetic Field

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Abstract

An analysis of the field concept and its description in terms of field variables leads to the development of an Interaction Theory of the electromagnetic field with the following properties: (1) it is free of self-interaction terms; (2) the point charge model remains a viable conceptual element of the theory; (3) radiative reaction is fully accounted for within the framework of the theory and is not an *ad hoc* addition as in conventional theory; (4) neither a single charged particle nor a system of interacting charged particles radiate to a mathematical sink at infinity. Radiation occurs only between and among the interacting particles. Energy and momentum are transferred only between and among the particles; (5) conventional conservation laws and forces are generalized; and (6) new conservation laws and forces appear. An application to a system of two interacting particles reveals in detail the conservation of energy, linear and angular momentum. Moreover, an intrinsic angular momentum of constant magnitude—a sort of classical helicity—appears. The significance of these results to the further development of electromagnetic theory and quantum theory is briefly discussed.

1. *Introduction*

In an earlier publication (Schwebel, 1970a), it was shown that the formulation of gravitation and electromagnetism as field theories places certain constraints on the mathematical formalism used to represent the physical concepts. The aim of the present report is to develop the previous study by concentrating on the electromagnetic field. For those readers already familiar with the earlier work, the extension to the gravitational field or any other field similarly structured will present no difficulties.

Our first concern will be an analysis of the field concept and the associated field variables. We will find that such difficulties as the infinite self-energy of a point charge, self-force, self-acceleration, etc., do not arise if the field variables are carefully defined. Moreover, the next section, which contains the derivations of the conservation laws for energy and momentum, will show why such self-interaction terms appear in conventional theory but not in the present theory. The following section reviews the mathematical formalism which the present theory requires. It also supplies those solutions for the field variables which satisfy the conditions imposed by Interaction Theory. The fifth section contains a discussion of a system of interacting charged particles. Here our analysis of the field will lead to results such as (i) that radiation occurs between and among the particles of the system; and (ii) that a single charged particle neither radiates nor stores energy in the field but that these events are properties of two or more particles. For this reason, we designate the present approach to field theory as an interaction theory. Whereas conventional field theories attribute the experimental data to one or another constituent in an experiment, Interaction Theory stresses the relational aspects of that data. The point-of-view taken is that any experimental data is joint property: it is representative of the behavior of all the constituents in an experiment and not the particular properties of one of them. The sixth section applies the theory to two interacting point sources. We see in detail the consequences of the general theory and the role that the new conservation laws and forces play in this special case. Finally, we discuss the work that remains to be done and the significance of what has been done for electromagnetic theory and quantum theory.

2. *The Field Concept*

Coulomb's law of force between two static charged particles, q_1 and q_2, is expressed by the relation

$$\mathbf{F}(q_1 \rightarrow q_2) = \frac{q_1 q_2}{r^3}\mathbf{r} \tag{2.1}$$

in which $\mathbf{r} = \mathbf{r}_2 - \mathbf{r}_1$ and $r = |r_2 - r_1|$, where $\mathbf{r}_1$ and $\mathbf{r}_2$ are the displacement vectors of the charges q_1 and q_2, respectively. The symbol $\mathbf{F}(q_1 \rightarrow q_2)$ expresses the force exerted on q_2 by q_1.

The field concept arises from the observation that the region about one of the charges, say q_1, can be explored by using the second charge, q_2, as a test charge. If, at each position of q_2 relative to q_1, we associate a vector whose magnitude and direction is equal to that of the force exerted on q_2 by q_1, then the mapping of the region about q_1, obtained by this procedure is said to exhibit the field about its source q_1. If the test charge is a unit charge, then the mapping is that of the *intensity* of the field which is usually symbolized by the letter **E** and, for purposes of brevity, called the electric field.

The mathematical representation of **E** follows from Coulomb's law and is

$$\mathbf{E}(q_1) = \frac{q_1}{r^3}\mathbf{r} \tag{2.2}$$

where **r** now denotes the displacement vector of an arbitrary spatial location relative to the source of the field q_1. The field variable, $\mathbf{E}(q_1)$, contains an explicit reference to its source and, as we see in equation (2.2), does not depend on any charge other than its source. It is that property that frees it for application over the entire region or field about q_1.

We can express Coulomb's law using the field variable **E**. Thus, we have

$$\mathbf{F}(q_1 \rightarrow q_2) = q_2 \mathbf{E}(q_1, q_2)$$

in which we have written the field variable as $\mathbf{E}(q_1,q_2)$ to indicate that the electric field is to be evaluated at the position of q_2 relative to its source q_1.

The reason for such a detailed presentation of a standard definition is that there is an aspect of the field concept which has not been fully appreciated, and consequently not consistently applied. Namely, the definition precludes the possibility of the source q_1 of the field acting both as a source and a test charge exploring the field about that source. Of course, we can use a second charge whose magnitude is equal to q_1 as a test charge, but that causes no difficulties. What is being denied is that the source of a field can also serve as a test charge for exploring and mapping that field. It follows that we can attach no meaning to the concept of an electric field acting on its source or vice versa. Such interactions as self-force, self-energy or any other self-interactions are not physically acceptable; they all represent interactions between a source and its own field.

Unfortunately, mathematical representations of these undesirable entities are possible so that we must devise a notation which excludes such meaningless terms. We achieve this by appending to each field variable a superscript which identifies the source of that field and by stipulating that any mathematical representation of an interaction between particles must bear distinct superscripts. For example, Coulomb's law between two charged particles, e^p and e^q should be written

$$\mathbf{F}(q \rightarrow p) = \frac{e^p e^q}{r^3}\mathbf{r} = e^p \mathbf{E}^q \tag{2.3}$$

in which $\mathbf{r} = \mathbf{r}^p - \mathbf{r}^q$ is the displacement vector between the two charged particles and $\mathbf{E}^q$ represents the electric field which is evaluated at the position of the pth particle but whose source is the qth particle.

We can repeat the same analysis for the magnetic field, almost verbatim, except that instead of charged particles we would employ current elements for the source terms and replace Coulomb's law of force between the static charges with Ampere's law of force between current elements. The same need to identify the magnetic field variable, **H**, with its source arises and a similar notation to that used for the electric field must be employed.

These alterations in notation brought about by the analysis of the field variable play no essential role when we turn to the task of establishing Maxwell's electromagnetic equations within the context of Interaction Theory. We follow the same procedure as in conventional theory and obtain the set of equations:

$$\nabla . \mathbf{E}^p = 4\pi\rho^p, \qquad (c = 1) \tag{2.4a}$$

$$\nabla . \mathbf{H}^p = 0, \qquad (c = 1) \tag{2.4b}$$

$$\nabla \times \mathbf{E}^p + \dot{\mathbf{H}}^p = 0, \qquad (c = 1) \tag{2.4c}$$

$$\nabla \times \mathbf{H}^p - \dot{\mathbf{E}}^p = 4\pi\mathbf{j}^p, \qquad (c = 1) \tag{2.4d}$$

These equations, though similar in form to Maxwell's equations, differ from them more significantly than the slight change in notation would seem to indicate. First, if the source $(\rho^p, \mathbf{j}^p)$ is absent then there can be no corresponding field variables $(\mathbf{E}^p, \mathbf{H}^p)$, and vice versa. It follows that solutions to the homogeneous equations, which are mathematically possible, are not physically meaningful. They would represent field variables which had no source in contradiction to the definition of a field variable. The mathematical problem of determining the physically meaningful solution to Maxwell's equations has been solved (Schwebel, 1970b). We will need the results for what follows, but for the details the original article should be consulted.

Secondly, the necessity of labeling each field variable with its source means that there is a set of equations for each source. The significance of this will become apparent when we turn to the derivation of the conservation laws and the equations of motion for any number of interacting charged particles.

Finally, Maxwell's equations [equations (2.4a)–(2.4d)], as a set which determines the field variables, $(\mathbf{E}^p, \mathbf{H}^p)$, in terms of the source, $(\rho^p, \mathbf{j}^p)$, and vice versa, are tautological. They are without physical content. Their function is a purely mathematical one of formulating a field description to replace an equivalent particle (source) description, or vice versa. Only when Maxwell's equations are coupled to Newton's laws of motion do we obtain a system of equations with physical content. A consequence of the tautological nature of Maxwell's equations is that the wave-particle dualism which plays so prominent a part in modern physics is seen to be the result of using two distinct, but equivalent, mathematical representations for the same physical entity. In other words, the right-hand sides of equations (2.4a)–(2.4d) are in terms of a particulate representation for the source of the electromagnetic field, whereas the left-hand sides are an equivalent formulation in terms of field variables.

We now turn to the derivation of conservation laws and equations of motion to illustrate and support the conclusion drawn from the form of Maxwell's equations in Interaction Theory.

3. *Conservation Laws*

The derivation of the conservation laws is almost identical in procedure to that used in conventional theory. The difference lies in the care that must be taken to ensure that a source is not acted upon by its own field. To derive the conservation of energy relation, we scalar multiply equation (2.4c) with $\mathbf{H}^q$. To this result, we add the equation obtained by interchanging p and q, to find

$$\partial_t(\mathbf{H}^p . \mathbf{H}^q) + \mathbf{H}^q . \nabla \times \mathbf{E}^p + \mathbf{H}^p . \nabla \times \mathbf{E}^q = 0$$

Next, we scalar multiply equation (2.4d) with $\mathbf{E}^q$, interchange p and q and then add the two results together. This yields

$$\partial_t(\mathbf{E}^p . \mathbf{E}^q) - \mathbf{E}^q . \nabla \times \mathbf{H}^p - \mathbf{E}^p . \nabla \times \mathbf{H}^q = -4\pi(\mathbf{j}^p . \mathbf{E}^q + \mathbf{j}^q . \mathbf{E}^p)$$

Finally, adding the above equations, we have the conservation relation

$$\partial_t(\mathbf{E}^p.\mathbf{E}^q + \mathbf{H}^p.\mathbf{H}^q) + \nabla.(\mathbf{E}^p \times \mathbf{H}^q + \mathbf{E}^q \times \mathbf{H}^p) = -4\pi(\mathbf{j}^p.\mathbf{E}^q + \mathbf{j}^q.\mathbf{E}^p) \quad (3.1)$$

Note, that if we dispense with the superscripts p and q, we obtain the result of conventional theory. As in that theory, we identify the energy density, w, with the expression

$$w = \frac{1}{4\pi}(\mathbf{E}^p.\mathbf{E}^q + \mathbf{H}^p.\mathbf{H}^q) \quad (3.2)$$

and the Poynting vector, $\mathbf{S}$, with

$$\mathbf{S} = \frac{1}{4\pi}(\mathbf{E}^p \times \mathbf{H}^q + \mathbf{E}^q \times \mathbf{H}^p) \quad (3.3)$$

The above relations were established for two interacting charged particles, but it can be extended without any difficulty to any number of interacting charged particles.

$$\partial_t\left\{\sum_{p \neq q}(\mathbf{E}^p.\mathbf{E}^q + \mathbf{H}^p.\mathbf{H}^q)\right\} + \nabla.\left\{\sum_{p \neq q}(\mathbf{E}^p \times \mathbf{H}^q + \mathbf{E}^q \times \mathbf{H}^p)\right\} = -4\pi \sum_{p \neq q}\{\mathbf{j}^p.\mathbf{E}^q + \mathbf{j}^q.\mathbf{E}^p\} \quad (3.4)$$

The summations are extended over all particles p and q, except $p \neq q$.

Observe that equations (3.2) and (3.4) require a minimal system of two interacting charges to give physical meaning to the quantities related. Thus, in Interaction Theory, a single charged particle can neither radiate nor store energy in the field. These quantities only appear when two or more charged particles interact.

Consider the total energy in the field of two static charges, q_1 and q_2, that are separated by a distance d. In this case,

$$\mathbf{E}^p = \frac{q_1(\mathbf{r} - \mathbf{r}_1)}{|\mathbf{r} - \mathbf{r}_1|^3}; \qquad \mathbf{E}^q = \frac{q_2(\mathbf{r} - \mathbf{r}_2)}{|\mathbf{r} - \mathbf{r}_2|^3}$$

where $\mathbf{r}_1 + \mathbf{d} = \mathbf{r}_2$. Of course, $\mathbf{H}^p = \mathbf{H}^q = 0$. An elementary calculation yields

$$\frac{1}{4\pi}\int d\tau \mathbf{E}^p.\mathbf{E}^q = \frac{q_1 q_2}{d}$$

Conventional theory, on the other hand, calculates the integral

$$\frac{1}{8\pi}\int d\tau(E^{p^2} + E^{q^2} + 2\mathbf{E}^p.\mathbf{E}^q)$$

The first two terms in the integrand are the self-energy contributions which, on integration, are infinite. The last term in the integrand is the term we evaluated above, and the only term which Interaction Theory gives.

It is clear why conventional theory requires a 'subtraction' procedure in order to make it 'work.' The failure to recognize the constraints imposed on a field variable has led to the inadvertent introduction of physically meaningless terms by the mathematical formalism of conventional theory.

It will prove convenient for what follows to write Maxwell's equations [(2.4a)–(2.4d)] in well-known tensor notation.

$$\partial_\mu F^{\mu\nu}(p) = 4\pi J^\nu(p) \tag{3.5a}$$

$$\partial_{[\mu} F_{\nu\lambda]}(p) = 0 \tag{3.5b}$$

Another useful representation is in terms of the dual of F^ν, i.e., $\hat{F}^{\mu\nu} = \frac{1}{2}\epsilon^{\mu\nu\lambda\rho}F_{\lambda\rho}$;

$$\partial_\mu \hat{F}^{\mu\nu}(p) = 0 \qquad (c=1) \tag{3.6a}$$

$$\partial_{[\mu} \hat{F}_{\nu\lambda]}(p) = 4\pi\epsilon_{\mu\nu\lambda\rho}J^\rho(p) \qquad (c=1) \tag{3.6b}$$

For purposes of reference, we have

$$F^{\mu\nu} = \begin{pmatrix} 0 & -E_x & -E_y & -E_z \\ E_x & 0 & -H_z & H_y \\ E_y & H_z & 0 & -H_x \\ E_z & -H_y & H_x & 0 \end{pmatrix}$$

$$F^{\mu\nu} = -F^{\nu\mu}$$

$$\hat{F}^{\mu\nu} = \begin{pmatrix} 0 & -H_x & -H_y & -H_z \\ H_x & 0 & E_z & -E_y \\ H_y & -E_z & 0 & E_x \\ H_z & E_y & -E_x & 0 \end{pmatrix}$$

with $\epsilon^{\mu\nu\lambda\rho}$ the well-known antisymmetric unit pseudotensor with $\epsilon^{0123} = +1$. The metric tensor $g^{\mu\nu}$ is taken to be

$$g^{\mu\nu} = \begin{pmatrix} 1 & & & \\ & -1 & & \\ & & -1 & \\ & & & -1 \end{pmatrix}$$

and $x^\mu = (x^0, x^1, x^2, x^3) = (t, x, y, z)$. In order to obtain a simple notation, we have omitted displaying the dependence of the field variables on the space and time coordinates. Instead, we have used that position to designate the source of the field variable.

We now proceed to derive the conservation laws in accordance with Interaction Theory. Not only will we derive a generalization of such laws as found in conventional theory, but we will obtain new ones which have no counterpart in that theory.

From equation (3.5a), we obtain

$$F_{\lambda\nu}(q)\,\partial_\mu F^{\mu\nu}(p) = 4\pi F_{\lambda\nu}(q)J^\nu(p)$$

which can be rewritten

$$\partial_\mu\{F_{\lambda\nu}(q)F^{\mu\nu}(p)\} - F^{\mu\nu}(p)\,\partial_\mu F_{\lambda\nu}(q) = 4\pi F_{\lambda\nu}(q)J^\nu(p)$$

Using equation (3.5b), we find, after some regrouping and the use of equation (3.5a),

$$\partial_\mu\{F^{\mu\nu}(p)F_{\lambda\nu}(q) - \tfrac{1}{2}\delta^\mu{}_\lambda F^{\alpha\beta}(p)F_{\alpha\beta}(q)\} - \tfrac{1}{2}F_{\nu\mu}(q)\,\partial_\lambda F^{\mu\nu}(p) = 4\pi F_{\lambda\nu}(q)J^\nu(p)$$

Next, interchange p and q in the above and add the resulting equation to it. In this way we obtain the first of the conservation laws.

$$\partial_\mu\{F^{\mu\nu}(p)F_{\lambda\nu}(q)+F^{\mu\nu}(q)F_{\lambda\nu}(p)-\tfrac{1}{2}\delta^\mu{}_\lambda F^{\alpha\beta}(p)F_{\alpha\beta}(q)\}$$
$$=4\pi\{F_{\lambda\nu}(q)J^\nu(p)+F_{\lambda\nu}(p)J^\nu(q)\} \quad (3.7)$$

It is a simple matter to verify that the equation for $\lambda=0$ is equation (3.1). The equations for $\lambda=1, 2, 3$ yield the conservation laws for the momenta densities which reduce to those found in conventional theory (Heitler, 1954) if the distinction imposed by p and q is removed.

If we start with equation (3.6b), we can form the relation

$$F^{\mu\nu}(q)\partial_{[\mu}\hat{F}_{\nu\lambda]}(p)=4\pi\epsilon_{\mu\nu\lambda\rho}J^\rho(p)F^{\mu\nu}(q)=8\pi\hat{F}_{\lambda\rho}(q)J^\rho(p)$$

Or,

$$F^{\mu\nu}(q)\{\partial_\mu\hat{F}_{\nu\lambda}(p)+\partial_\nu\hat{F}_{\lambda\mu}(p)+\partial_\lambda\hat{F}_{\mu\nu}(p)\}$$
$$=\partial_\mu\{F^{\mu\nu}(q)\hat{F}_{\nu\lambda}(p)\}+\partial_\nu\{F^{\mu\nu}(q)\hat{F}_{\lambda\mu}(p)\}+\partial_\lambda\{F^{\mu\nu}(q)\hat{F}_{\mu\nu}(p)\}$$
$$-\hat{F}_{\nu\lambda}(p)\partial_\mu F^{\mu\nu}(q)-\hat{F}_{\lambda\mu}(p)\partial_\nu F^{\mu\nu}(q)-\hat{F}_{\mu\nu}(p)\partial_\lambda F^{\mu\nu}(q)=8\pi\hat{F}_{\lambda\rho}(q)J^\rho(p)$$

Whence, using equation (3.5a),

$$\partial_\mu\{F^{\mu\nu}(q)\hat{F}_{\nu\lambda}(p)+\hat{F}^{\mu\nu}(p)F_{\nu\lambda}(q)+\tfrac{1}{2}\delta^\mu{}_\lambda F^{\alpha\beta}(q)\hat{F}_{\alpha\beta}(p)\}$$
$$=4\pi\{\hat{F}_{\lambda\rho}(q)J^\rho(p)-\hat{F}_{\lambda\rho}(p)J^\rho(q)\} \quad (3.8)$$

In the above, we used the relation that

$$\hat{F}_{\mu\nu}(p)\partial_\lambda F^{\mu\nu}(q)=\hat{F}^{\mu\nu}(p)\partial_\lambda F_{\mu\nu}(q)=\hat{F}^{\mu\nu}(p)\{-\partial_\mu F_{\nu\lambda}(q)-\partial_\nu F_{\lambda\mu}(q)\}$$
$$=-\partial_\mu\{\hat{F}^{\mu\nu}(p)F^{\nu\lambda}(q)\}-\partial_\nu\{\hat{F}^{\mu\nu}(p)F_{\lambda\mu}(q)\}$$
$$=-2\partial_\mu\{\hat{F}^{\mu\nu}(p)F_{\nu\lambda}(q)\}$$

The second step is a consequence of equation (3.6a).

The conservation laws expressed by equation (3.8) are dependent on the constraint that the p, q notation imposes. For, if this distinction is removed each term vanishes identically; the right-hand side obviously does and we can prove that it is so for the left-hand side.

Consider,

$$F^{\mu\nu}(q)\hat{F}_{\nu\lambda}(p)=\tfrac{1}{2}\epsilon^{\mu\nu\alpha\beta}\hat{F}_{\alpha\beta}(q)\tfrac{1}{2}\epsilon_{\nu\lambda\sigma\tau}F^{\sigma\tau}(p)$$
$$=-\tfrac{1}{2}\delta^\mu{}_\lambda\hat{F}_{\alpha\beta}(q)F^{\alpha\beta}(p)+\hat{F}_{\lambda\nu}(q)F^{\mu\nu}(p)$$

The last line results from the evaluation of the product of the pseudotensors. In the same way, we can establish that

$$\hat{F}^{\mu\nu}(p)F_{\nu\lambda}(q)=-\tfrac{1}{2}\delta^\mu{}_\lambda F_{\alpha\beta}(p)\hat{F}^{\alpha\beta}(q)+F_{\lambda\nu}(p)\hat{F}^{\mu\nu}(q)$$

With these relations, we can write equation (3.8)

$$\partial_\mu\{F^{\mu\nu}(q)\hat{F}_{\nu\lambda}(p)-F^{\mu\nu}(p)\hat{F}_{\nu\lambda}(q)+\hat{F}^{\mu\nu}(p)F_{\nu\lambda}(q)-\hat{F}^{\mu\nu}(q)F_{\nu\lambda}(p)\}$$
$$=8\pi\{\hat{F}_{\lambda\rho}(q)J^\rho(p)-\hat{F}_{\lambda\rho}(p)J^\rho(q)\} \quad (3.9)$$

In this form, it is apparent that when $p=q$ each term vanishes identically.

Although the conservation laws have been derived for two interacting sources, their extension to any number of interacting sources follows immediately.

Conventional electromagnetic theory designates equations (3.7), (3.8) and (3.9) as conservation laws. Interaction Theory, although it can reproduce the analogous relations which accepted theory develops, leads to a more profound interpretation. Because Maxwell's equations are tautologies, equations (3.7), (3.8) and (3.9), which are purely mathematical consequences from them, are likewise tautological. The left-hand sides of these equations are the equivalent field representations of their right-hand sides. Specifically, equation (3.1) is one such relation for two interacting sources. (We could consider the more general case of n interacting particles, but the essentials are unaltered and the mathematics is only changed by a summation sign.) For point sources, the right-hand side of equation (3.1) when integrated over all space represents the energy per unit time which particle q expends on p added to that which particle p expends on q. The left-hand side of this equation expresses precisely the same physical content, but described in terms of the appropriate field variables. We will show below that the divergence term integrated over all space vanishes, so that we are left with the contribution from the remaining term which is the rate-of-change of the energy as expressed in terms of the appropriate field variables. We can say more, but before doing so we must review the solutions for the field variables which satisfy the conditions imposed by Interaction Theory.

4. *Solutions for the Field Variables*

The equations which must be solved to determine the field variables follow from equations (3.5a) and (3.5b). From equation (3.5b), we form

$$\partial^{\mu}\partial_{[\mu}F_{\nu\lambda]} = 0 = \partial^{\mu}\{\partial_{\mu}F_{\nu\lambda} + \partial_{\nu}F_{\lambda\mu} + \partial_{\lambda}F_{\mu\nu}\}$$

or, using equation (3.5a), we have

$$\Box F_{\nu\lambda} \equiv \partial^{\mu}\partial_{\mu}F_{\nu\lambda} = 4\pi\{\partial_{\nu}J_{\lambda} - \partial_{\lambda}J_{\nu}\} \tag{4.1}$$

Interaction Theory requires that the absence of a source implies the absence of the corresponding field variable. It follows that in equation (4.1), if the right-hand side of the equation is zero for all space-time, i.e., no source is present, then $F_{\nu\lambda}$ must be identically zero. In order that this be the case, we must require that the inverse of the D'Alembertian, $\Box^{-1}$, exists.

The mathematical solution to this problem has been given (Schwebel, 1970b). It was shown that if

$$\Box\psi(\mathbf{r}, t) = g(r, t)$$

then the solution we seek is given by

$$\psi(\mathbf{r}, t) = \frac{1}{4\pi}\int \frac{d^3x'}{R}\{H(t-R)g(\mathbf{r}', t-R) + H(-t-R)g(\mathbf{r}', t+R)\}, \qquad (c=1)$$

with $R = |\mathbf{r} - \mathbf{r}'|$ representing the distance between the field point $\mathbf{r}$ and that portion of the source located at $\mathbf{r}'$. The Heaviside function is defined by

$$\begin{aligned} H(t) &= 1, \qquad (t > 0) \\ &= 0, \qquad (t < 0) \end{aligned}$$

The significance of this result for us is that for $|t| < R$, the field variable, $\psi(\mathbf{r}, t)$, vanishes. Therefore, for a system of interacting particles at some time t, it is clearly possible to enclose the entire system within a surface on which and external to which the field variables vanish. Evidently, there is no transfer of radiation through such a surface. This result justifies the statement made at the close of the last section.

We can apply the above result to the 'conservation laws' [equations (3.7), (3.8) and (3.9)], for any number of interacting sources. The right-hand sides of these equations describe the interactions among the various sources at those sources. It follows that the left-hand side of these equations describes precisely the same physical events, although with a different set of variables. Consequently, there can be no losses or radiation from such a system to some mathematical sink at infinity—the surface over which and external to which the field variables vanish assures us of this. Radiation occurs between and among interacting sources—it will leave a system provided there are interacting sources outside that system.

5. *Equations of Motion*

The interpretation given to the left-hand side of equation (3.7) depends on the role it plays in Newton's equations of motion. In standard notation, these are

$$\frac{d}{dt} p_\lambda(q) = \sum_{k \neq q} \int d\tau_q \{F_{\lambda\nu}(k) J^\nu(q)\} \tag{5.1}$$

in which the integration is over a volume element which contains only the point source q and the summation is over all the other sources with which it interacts.

If we integrate equation (3.7) over the volume $d\tau_q$ and use the result in the above equation, we obtain

$$\frac{d}{dt}\left[p_\lambda(q) - \sum_{k \neq q} \frac{1}{4\pi} \int d\tau_q \{F^{0\nu}(k) F_{\lambda\nu}(q) + F^{0\nu}(q) F_{\lambda\nu}(k) - \tfrac{1}{2}\delta_\lambda{}^0 F^{\alpha\beta}(k) F_{\alpha\beta}(q)\}\right]$$

$$= \sum_{k \neq q} \frac{1}{4\pi} \int d\tau_q \, \partial_l \{F^{l\nu}(k) F_{\lambda\nu}(q) + F^{l\nu}(q) F_{\lambda\nu}(k) - \tfrac{1}{2}\delta_\lambda{}^l F^{\alpha\beta}(k) F_{\alpha\beta}(q)\} \tag{5.2}$$

with $l = 1, 2, 3$. According to Interaction Theory and the mathematical procedure pursued, we have replaced equation (5.1) by an equivalent one in which field quantities are exhibited, and the role they play can be identified. Thus, the term containing the integral on the left can, in obvious imitation of its relation to the mechanical four-momentum, be called the momentum associated with the field and its integrand defined to be the density of the field momentum in the neighborhood of the source q. The integral of the term on the right is clearly a divergence of a quantity which can readily be compared to the flow of the four-momentum density through the surface surrounding the source q. The latter does not vanish in this instance, because the surface does not enclose all the interacting sources. What is being represented in the neighborhood of the source q is the four-momentum delivered to or removed from it by the other interacting sources plus that portion of it that is in the process of being transferred to or from that region through the surface surrounding the source q. In fact, we get a detailed picture of what the field is doing in the neighborhood of a source; a mathematical representation of Faraday's notions about the region surrounding a charged source. We see that radiation of energy and momentum from a source is to or from other sources which interact with it.

What occurs in the absence of other interacting systems can be determined from equation (5.1) as well. Let us sum that equation over all the interacting sources.

$$\frac{d}{dt}\sum_q p_\lambda(q) = \sum_q \sum_{k\neq q} \int d\tau_q \{F_{\lambda\nu}(k) J^\nu(q)\} \tag{5.3a}$$

$$= \sum_q \sum_{k\neq q} \int d\tau \{F_{\lambda\nu}(k) J^\nu(q)\} \tag{5.3b}$$

$$= \frac{1}{2}\sum_q \sum_{k\neq q} \int d\tau \{F_{\lambda\nu}(k) J^\nu(q) + F_{\lambda\nu}(q) J^\nu(k)\} \tag{5.3c}$$

In the second equation, we have replaced the volumes of integration $d\tau_q$ about each source with a volume $d\tau$ over all space. This can be done because the sources are point charges. The third equation is merely an obvious symmetrization, which is possible because the summation is over all the sources.

If we sum equation (3.7) over all the interacting sources in the system and integrate the result over all of space, then the mathematical results given in section four show that there are no losses over the surface of the enclosing volume; i.e., no losses to some mathematical sink at infinity. Thus, the right-hand side of equation (5.3c) is related directly to the time component of equation (3.7) and we obtain a conservation law:

$$\frac{d}{dt}\left[\sum_q p_\lambda(q) - \frac{1}{8\pi}\sum_q \sum_{k\neq q} \int d\tau \{F^{0\nu}(k) F_{\lambda\nu}(q) + F^{0\nu}(q) F_{\lambda\nu}(k) - \tfrac{1}{2}\delta_\lambda{}^0 F^{\alpha\beta}(k) F_{\alpha\beta}(q)\}\right] = 0 \tag{5.4}$$

This relation is the mathematical form of the statement that the complete system of interacting sources does not radiate to some mathematical sink at infinity. If there were sources outside the system with which the latter interacts, then equation (5.2) summed over that system or over the outside sources would be applicable. If such were the case, then radiation would occur between those sources and the system.

Herein lies a major difference between Interaction Theory and conventional Maxwell Theory. In the latter, a system radiates and, in general, loses energy. In order to account for this loss, the concept of radiative reaction is introduced. The difficulties and complications that arise as a consequence of this notion are too well known to need detailed exposition here. The source of these problems stems from the evaluation of Poynting's vector for the self-interaction between a source and its field variables (Heitler, 1954). Interaction Theory does not contain such quantities, but it does retain that aspect of radiative reaction which is physically sound. In Interaction Theory, if there is radiation it occurs between or among the interacting sources of the system. Consequently, the motion of the sources is changed, and that is specifically what is meant by radiative reaction. For, as we have seen, it is the motion of the sources which reveals the physical behavior of the system.

So far we have dealt with equation (3.7) which has a counterpart in conventional theory. What can be said about equation (3.8) where no similar connection can be exploited?

The answer has been sought by exploring the effect of the dual terms in similar mechanical problems. In an earlier article (Schwebel, 1971), it was shown that if we treat the dual of the Lorentz-force term, i.e., the terms on the right-hand side of equation (3.8), as additional forces on the sources, then we obtain the same result for the orbit of one particle gravitating about another, as given by the General Theory of Relativity. In the light of this result, a tentative proposal for interpreting equation (3.8) is to form a linear combination with equation (3.7) and treat the augmented 'conservation equation' as we have equation (3.7). Since all the operators are linear, there are no essential difficulties in following through such a procedure. We will not pursue the general theory further but apply the results we have obtained to an analysis of the interaction between two charged point sources. The treatment by conventional theories of this problem has led to well-known complications, and we will consider the resolution of these difficulties as given by Interaction Theory.

6. *Two-Body Problem*

The system under study is to consist of two interacting particles with charges e^p and e^q, respectively. From a relational mechanics point of view (Schwebel, 1970c) one of the particles, say the pth, is the fiducial system for the position and momentum of the qth particle. The equations of motion are, from equation (5.1),

$$\frac{d}{dt}\mathbf{p}(q) = e^q \mathbf{E}^p - \lambda e^q \mathbf{v}^q \times \mathbf{E}^p, \qquad (c = 1) \tag{6.1a}$$

$$\frac{d}{dt}p^0(q) = e^q \mathbf{v}^q . \mathbf{E}^p, \qquad (c = 1) \tag{6.1b}$$

We have used the fact that the qth particle is moving in the static field of the pth particle, i.e., $\mathbf{v}^p = 0$ and $\mathbf{H}^p = 0$.

Employing the relativistic values for $\mathbf{p}$ and $\mathbf{p}^o$, setting

$$\mathbf{E}^p = \frac{e^p}{r^3}\mathbf{r}$$

and letting μ be the reduced mass, in accordance with relational mechanics, the above equations become

$$\frac{d}{dt}\left\{\frac{\mu\mathbf{v}}{(1-v^2)^{1/2}}\right\} = \frac{e^q e^p}{r^3}\mathbf{r} - \lambda e^q e^p \frac{\mathbf{v}\times\mathbf{r}}{r^3} \tag{6.2a}$$

$$\frac{d}{dt}\left\{\frac{\mu}{(1-v^2)^{1/2}}\right\} = e^p e^q \frac{\mathbf{v}\cdot\mathbf{r}}{r^3} = -e^p e^q \frac{d}{dt}\left(\frac{1}{r}\right) \tag{6.2b}$$

since $\mathbf{v} = (d/dt)\mathbf{r}$.

The last equation integrates to yield

$$\frac{\mu}{(1-v^2)^{1/2}} + \frac{e^p e^q}{r} = E = \text{constant} \tag{6.3}$$

with E representing the total energy of the system.

If we take the cross-product of equation (6.2a) with r and simplify, we obtain on integration the relation

$$\frac{\mu\mathbf{r}\times\mathbf{v}}{(1-v^2)^{1/2}} + \lambda e^p e^q \frac{\mathbf{r}}{r} = \mathbf{L} = \text{constant} \tag{6.4}$$

which states that the total angular momentum of the system is constant.

The contribution of the dual force to the total energy of the system appears in the calculation of the magnitude of the velocity. However, its contribution to the angular momentum is much more significant. We find that it gives rise to an intrinsic angular momentum of constant magnitude and in a direction which lies along the displacement vector of the two interacting charged particles—a sort of helicity of classical origin.

Equations (6.3) and (6.4) can be solved for the trajectory of one of the particles about the other. In fact, the procedure and solution are essentially the same as that obtained for two particles interacting with one another through the gravitational field (Schwebel, 1971). The mathematical details of such a derivation is of minor interest at the moment. What is important is that Interaction Theory, in contradistinction to conventional Maxwell theory, concludes that a system of two or more charges is stable; there is no loss of either energy or angular momentum from such a system. Moreover, a heretofore exclusive property of quantum theory, spin, is seen to be an integral property of a classical theory. Note also that linear momentum is also conserved, since this must be the case in the formulation of Newton's laws of motion in accordance with relational mechanics (Schwebel, 1970c).

7. *Discussion*

A critical analysis of the concept of field and its description in terms of field variables has led in a natural way to the elimination of such concepts as self-energy, self-force or self-interactions of any type. We have explicitly shown in the case of the self-energy of a point charge how conventional theory leads to infinite values and why such unacceptable concepts and results appear.

The changes that our analysis makes in conventional theory leads to the formulation of an *Interaction Theory* which does not contain these unphysical consequences and concepts. In the new theory, the charged particle concept or, if one prefers, the point charge model, remains a viable conceptual element in electromagnetic theory. There are generalizations of the conservation laws and forces of conventional theory and also new conservation laws and forces which have no counterpart in that theory. Radiative reaction which is an *ad hoc* addition to conventional theory is an essential part of the present theory. It is shown that radiation occurs between and among the interacting systems, and determines the motion of the sources. It is the latter's response which is the reaction to radiation.

Besides the general analysis of an arbitrary system of interacting sources, a specific application of the theory to two interacting particles was presented. We were able to show in detail for this simplest of interacting systems, that energy and angular momentum are conserved. In particular, the existence of an intrinsic angular moment of constant magnitude—a sort of classical helicity—was revealed. What, heretofore, was considered to be specific to quantum theory arises within a classical context.

The last result raises the question of the quantum mechanical aspects of the new conservation laws and forces, indeed, of Interaction Theory itself. Some work along these lines has been done and will be submitted for publication.

For classical electromagnetic theory, Interaction Theory presents new insight into some old and perplexing problems. Conventional theory holds it to be impossible to formulate a Lagrangian for a system of interacting charged particles. The basis for this contention is the independence of the field; it has its own set of variables and exists independently of its source. There is no such field-particle dualism in Interaction Theory. The field and particle descriptions are merely two different mathematical models of the same physical entity. Consequently, there is no difficulty in constructing a Lagrangian formalism for interacting sources, and that possibility is significant as well for quantum mechanics and quantum field theory.

Two other consequences of similar importance follow from Interaction Theory. First, gauge transformations are not admissible, for they imply the presence of field variables in the absence of sources. Secondly, the mathematical formalism required by the theory provides a unique, well-defined propagator for electromagnetic interactions. The impact of these conclusions on quantum electrodynamics and field theory in general will be the subject of future publications.

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